United States Naval Postgraduate School

THESIS

ON SURVIVABILITY OF COMMUNICATION NETWORKS

by

Seyfettin Enginler

Thesis Advisor: S. G. Chan

December 1971

Approved for public release; distribution unlimited.

61

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D		
(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)		
1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP	
3. REPORT TITLE ON SURVIVABILITY OF COMMUNICATION NETWORK		
4. DESCRIPTIVE NOTES (Type of report and, inclusive dates) Master's Thesis; December 1971		
5. AUTHOR(S) (First name, middle initial, last name) Seyfettin Enginler Lieutenant, Turkish Navy		
6. REPORT DATE December 1971	7a. TOTAL NO. OF PAGES 61	7b. NO. OF REFS 12
8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
10. DISTRIBUTION STATEMENT Approved for public release; distribution unlimited.		
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940	

13. ABSTRACT

The probability of survival of a communication network is defined as the probability that there exist at least one path between any pair of stations within the network. In this thesis, four methods for the calculation of the probability of survival of the network, which is under enemy attack, are presented.

The first two methods deal with random networks whose links have finite and identical probability of survival, while the third and fourth methods are based on the min-cut max-flow theorem.

DD FORM 1 NOV 65 1473 (PAGE 1)
S/N 0101-807-6811

Unclassified
Security Classification

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Probability of Survival						
Min-cut Max-flow Theorem						
Enemy Attack						

On Survivability of Communication Networks

by

Seyfettin Enginler
Lieutenant, Turkish Navy
B.S., United States Naval Postgraduate School,
March 1971

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1971

Author Enginler

Approved by: S. G. Chan
Thesis Advisor

R Parker
Chairman, Department of Electrical Engineering

Milton U. Clauser
Academic Dean

ABSTRACT

The probability of survival of a communication network is defined as the probability that there exist at least one path between any pair of stations within the network. In this thesis, four methods for the calculation of the probability of survival of the network, which is under enemy attack, are presented.

The first two methods deal with random networks whose links have finite and identical probability of survival, while the third and fourth methods are based on the min-cut max-flow theorem.

TABLE OF CONTENTS

ACKNOWLEDGEMENT

The author wishes to express his appreciation to Professor S. G. Chan for the invaluable aid and counsel which he has offered during the preparation of this thesis and to Professor G. H. Marmont for reading the final manuscript.

I. INTRODUCTION

A communication network is a set of nodes connected by links. Every link has a branch capacity which indicates the maximum amount of flow of messages. A communication network must have large enough branch capacity such that all messages can reach their destinations under specified conditions. In general these message requirements vary with time.

A communication network may be considered as a collection of message centers that attempt to transfer information to one another over a variety of connecting channels. However, neither the centers nor the channels are necessarily survivable at any given time. For example in military applications a center might be destroyed by enemy attack, or lose its power supply. Likewise, a communication channel might be busy, or it might be inoperative because of an amplifier failure, a broken or cut telephone wire, or a jammed radio link. In spite of these possibilities, it is highly desirable that the remaining switching centers be able to communicate with each other.

A reasonable definition of survivability of a communication network is that there be at least one path between any pair of stations. The survivability of a military communication network is related to the exact structure of the network and the probability of survival of its links. It is also related to the enemy attack and the topology of the network.

If one wants to enhance the probability of survival of the network, he might increase the probability of survival of the links, or he might increase the number of links between pairs of stations without increasing the total probabilities of survival of the links or he might change the topology of the network or use some combinations of these techniques. The choice of techniques depends heavily on the cost of the network.

Communication links are made up of one or more elements such as cables, antennas, repeaters, or buildings which house the communication equipment.

The analysis of the survivability of the communication networks has been studied by various investigators [5], [6], [8] and [11]. In the work of E. Moore and C. Shannon [8], the probability of communication between given pair (x,y) of nodes in the network is investigated. In reference 5, the idea of the overall survivability of the finite communication network is introduced. Two formulas are given for calculating the overall probability of survival.

In this paper, random networks and finite networks whose links have a finite probability of survival under nuclear weapon attacks are considered. Four methods are given to calculate the probability of survival of the communication network. First two methods apply to the random networks; one of them is without the consideration of the length of the path between any pair of stations. The probability of survival of the finite networks are calculated by approximation methods using

the min-cut theorem. Last method gives accurate results for finite networks and is computationally feasible for networks with several thousand stations or nodes.

A. THE MATHEMATICAL MODEL

A communication network has n stations (or nodes) designated by $v_1,\ldots,v_n$, which are connected by links. Every station has an average of s number of links and no self loops. Also, all links are assumed identical with equal probability of survival.

A communication network might have fixed topological structure such as a microwave relay system, or might have time varying structure such as a nonsynchronous satellite communication network.

The stations are assumed to have high probability of survival. In this model of a communication network, the probability of survival is assumed to be unity.

The probability of survival of a link is related to the distance between a pair of links and the structure of the link. Also assumed is the separation between links of a network be ensured that one weapon will not destroy more than a predetermined number of links.

If the enemy wants to destroy a system, he can organize his attack in one of many ways. He can aim his weapons at all its series links, or he can aim at any portion of the network. The choice of his attack depends on probable location of the links, the degree of importance of the links, and energy level of his weapons, etc.

The following condition is assumed for enemy attack. The nuclear weapon is aimed at random into a region of area A. The probability that any given nuclear weapon is aimed at a region of area Δ is Δ / A ($\Delta \leq A$).

A nuclear weapon has many effects. In this thesis, the destructive effect is mentioned, which is due mainly to blast or shock damages to structures either through the crushing action of the peak overpressure, or through the lateral displacement, tumbling or tearing apart caused by the dynamic pressures. Also, the damage caused by a nuclear weapon is classified by degree as follows: [2]

Type A: Completely destroyed.

Type B: Damage severely and beyond repair.

Type C: Damage that requires major repairs.

Type D: Light damage.

The schematic illustration of distribution of the types of damages is shown in Fig. 1.

The average radius of damage is assumed to be R. Inside R_1, every link is completely destroyed and there is no damage outside the radius R_2 when nuclear explosion occurs at point (0,0). Thus, some links will not be destroyed and some links only partially destroyed, when the links are located between the radii R_1 and R_2. In this paper, it will be assumed that the probability of damage of links located between radii R_1 and R_2 follows the Gaussian distribution.

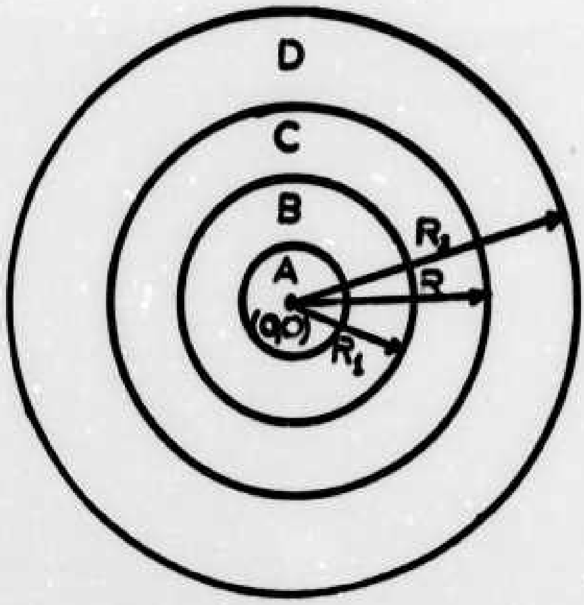


Figure 1

The diagram of distribution of the types of damages.

II. SURVIVABILITY OF A LINK UNDER NUCLEAR WEAPON ATTACK

When a nuclear weapon falls near a target at a distance less than the damage radius R_1 from a link, the link is always totally destroyed. However the network may still maintain communications.

If the distance between a pair of links is 2r miles and r is less than the damage radius R_1 for the nuclear weapon used by the enemy, the links may have high probability of damage or low probability of survival. The distance 2r must be at least twice the damage radius R_2 of the nuclear weapon in order to get a high probability of survival. This radius R_2 is a function of the yield of the nuclear weapon. If someone wants to design a communication network, he must estimate the size of the largest weapon of the enemy. For instance, if the explosion occurs above the surface of the ground or water, a 1 MT. nuclear weapon has a damage radius of about 10.5 miles and 20 MT. nuclear weapon has a 27 mile damage radius [2].

With the aid of a computer, the integration of the probability function over a known damage radius of the various MT. weapon is shown in Fig. 2. If a link is sufficiently distant from a given MT. explosion and the distance **r** lies well to the right of the applicable MT. curve in Fig. 2, the survival of that link is almost certain. If, on the other hand, the distance r lies to the left of the applicable curve, the destruction of the link is almost certain.

The probability of survival given by Fig. 2 is the probability that one nuclear weapon falls specified distance away from the link. If more than one nuclear weapon is aimed at different points of the communication network, the probability of survival of the link is the product of the probability of survival associated with each nuclear weapon. That is

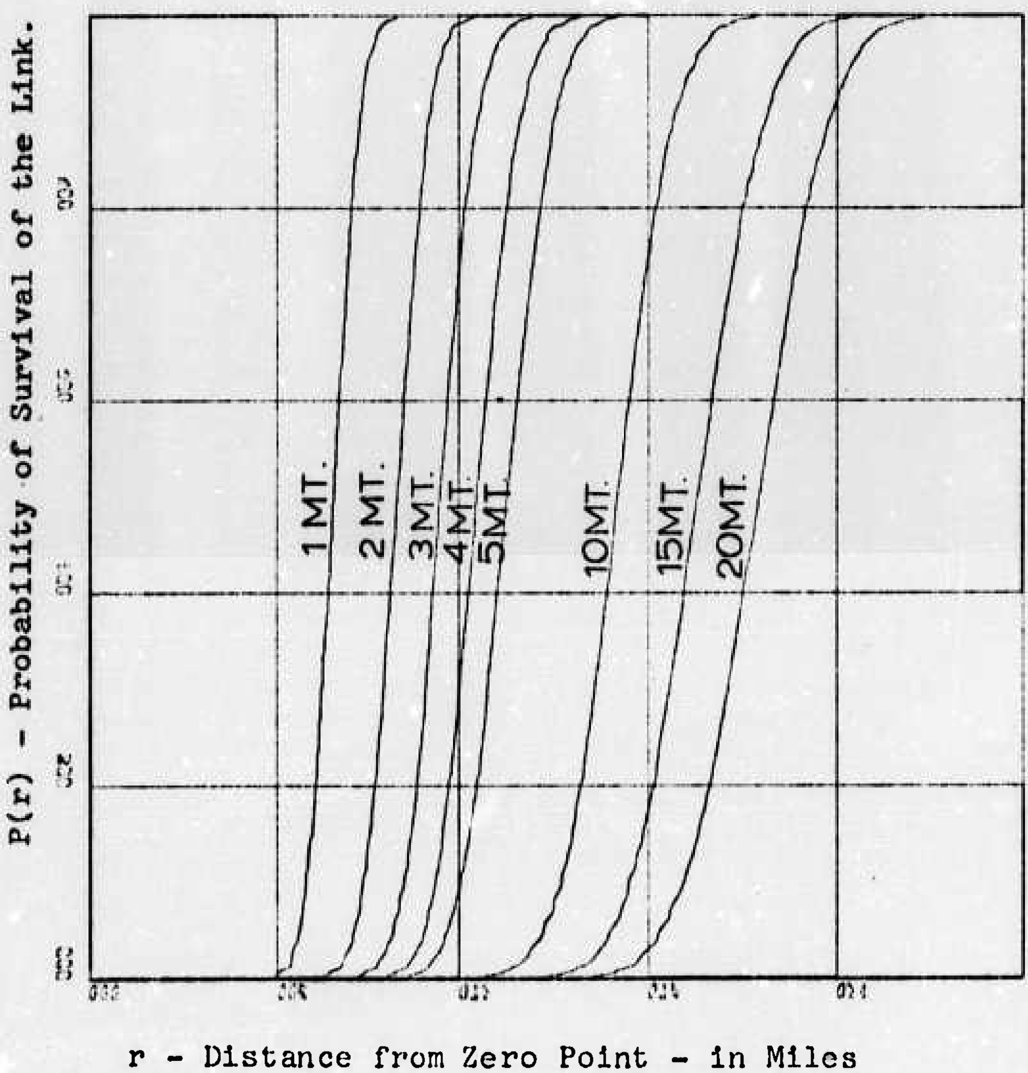


Figure 2

$$P(N) = p_1(x_1)\ p_2(x_2)\ \dots\ p_N(x_N) \tag{1}$$

where P(N) is the total probability of survival of the link under N nuclear weapon attacks. x_k is the distance from the zero point of the k^{th} nuclear weapon. $p_k(x_k)$ is the probability of survival as given in Fig. 2 for distance x_k at various energy levels of the weapon.

If the link is sufficiently far from the zero points of each weapon such that x_k lies well to the right of the associated curve for a given weapon, $p_k(x_k)$ is very close to unity and considerably greater than 0.99. If, for example, the number of nuclear weapons were 10, P(10) would still be 0.9. Thus, the value of P(10) is still near 1.0. Therefore, if the distance between zero points of each weapon and the link is far enough, the number of the nuclear weapons does not influence the probability of survival of the link.

EXAMPLE 1: Let three 5-MT. nuclear weapons be aimed at some area. The distance from the zero points to the link are 12, 15 and 18 miles respectively. What is the probability of survival of the link?

According to equation (1)

$$P(3) = \prod_{k=1}^{3} p_k(x_k)$$

$$P(3) = p_1(x_1)\ p_2(x_2)\ p_3(x_3)$$

where

$$x_1 = 12 \text{ miles}$$

$$x_2 = 15 \text{ miles}$$

$$x_3 = 18 \text{ miles}$$

From Fig. 2 for a 5 MT. nuclear weapon

$$p_1(12) = 0.09$$

$$p_2(15) = 0.895$$

$$p_3(18) = 0.999$$

So, the probability of survival of a link is 0.08.

In practice, it is too difficult to estimate or measure the distance between links and the zero points of each weapon for which the probability of survival of a link is computed. However, a communication network should always have more than one link and also the nuclear weapon can destroy more than one link.

Assuming 2r to be the average distance between each pair of links, we may now compute the average probability of survival of a link which is integrating over the area of radius r. Then,

$$p(r) = \int_0^r f(x)\, dx \qquad (2)$$

where f(x) is the Gaussian distribution function of survivability with mean and variance, and 2r is the average distance between links in miles.

When the communication network is subjected to random bombardment, the probability of survival of a link is a function of the average distance between links, but it is independent of its location. However, for one nuclear weapon, the probability of survival of a link is related to the ratio of the damage area of the nuclear weapon to the area which is subjected to random bombardment.

Various energy level nuclear weapons have different damage areas and different means and variances. Every various energy level nuclear weapon has a different damage distribution of probability.

The nuclear weapon is targeted at random into a region of area A square miles. The probability of a link being inside the damage area of the nuclear weapon is the ratio of the damage area of the nuclear weapon to an area which is subjected to random bombardment. Thus,

$$\frac{D}{A} \quad (D \leq A)$$

where D is the damage area and equal to πR_2^2, R_2 is the damage radius of the nuclear weapon.

The probability of damage of any given link which is inside this area is

$$Q = \frac{D}{A}[1 - p(r)]$$

where 1-p(r) is the probability of damage of a link which is 2r miles from other link or r miles from the zero point.

The probability of survival of any given link inside the area of A square miles is

$$P = 1 - Q \tag{3}$$

Equation (3) is valid for one nuclear weapon. If N nuclear weapons fall at random in an area of A square miles, the following is valid:

$$P(N) = \{1 - [1 - p(r)]\frac{D}{A}\}^N \tag{4}$$

EXAMPLE 2: Let 30 miles be the average distance between links. Three 5 MT. nuclear weapons are randomly aimed at an area of 1000 square miles. What is the probability of survival of any given link in this area?

The probability of survival of a link which is 15 miles from the zero point is

$$p(15) = 0.90$$

The damage radius of the 5 MT. nuclear weapon is 17 miles. Therefore, the damage area, D, is 907.5 square miles.

The probability of a link being in damage area, D, is

$$\frac{D}{A} = 0.9075$$

For one nuclear weapon, the probability of survival of any given link in area of A square miles is

$$P(1) = 0.989$$

For three nuclear weapons;

$$P(3) = 0.967$$

If the average distance between links is large enough, the number of nuclear weapons does not appreciable affect the probability of survival of the links.

If the average distance between a pair of links is 24 miles vice 30 miles, the probability of survival of any given link is changed drastically. Thus, from Fig. 2

$$p(12) = 0.10$$

The probability of any given link inside the damage area is unchanged and again equal to 0.9075.

For one nuclear weapon, the probability of survival of a link in same area of 1000 square miles is

$$P(1) = 0.183$$

For three nuclear weapons

$$P(3) = 0.0043$$

From the above examples, the average distance between links is the most critical factor, since it was shown that a change of only 6 miles or 20 percent resulted in a significant difference in the probability of survival varied from 98.9 percent to 0.43 percent.

III. RANDOM NETWORK

Let a communication network be an aggregate of stations and each station is capable of issuing some number s of links. Each link terminates at some station of the aggregate, and the probability that a link from one station terminates at another station is the same for every pair of stations. The resulting configuration is called "random communication network" [6].

A. THE SURVIVABILITY OF THE COMMUNICATION NETWORK WITHOUT CONSIDERING THE LENGTH OF THE PATH

In the last chapter, the probability of survival of any given link inside some area of A square miles is considered under various conditions based on the size of the enemy weapons and the distance between pairs of links.

The average number of links, s, alone cannot give precise information about the communication network survivability. In addition, the relationship between the average number of links and the probability of survival of the communication network is needed to determine the survivability of the network.

Markoff chains can be used to find the probability of survival of the random communication network. Suppose that an urn contains n balls with w white balls and n-w black balls and a player has s tickets. He plays one ticket for the right to draw a ball at random from the urn. If the ball drawn is white, he receives d additional tickets and if it is black he receives nothing. The ball drawn is always replaced by a black ball. Drawings continue until s=0. In this case, black

balls represent stations not reached previously, and tickets represent the number of links emanating from previously reached stations, which have not yet been traced.

Let H be the probability of survival of the communication network which is a function of the number of links.

$$H = f(s) \tag{5}$$

The average number of links after the enemy attack can be calculated as follows: Let d be the average number of links, after the enemy attack, d is equal to the average number of links before the enemy attack times the probability of survival of any given link inside some area of A square miles. Thus

$$d = s\,P(N) \tag{6}$$

where P(N) is the probability of survival of any given link inside some area of A square miles and s is the average number of links before the enemy attack.

The urn problem can also be applied to our random communication network. The existence of a path in a random communication network from a station v_i to a station v_j implies the possibility of tracing links from v_i through any number of intermediate stations to v_j.

v_j is m links removed from v_i, if m is the smallest number of links contained in any of the paths from v_i to v_j. Station v_i itself is zero link removed from v_i. All the other stations upon which the links of v_i terminate are one link removed. The stations upon which the links from these latter stations

terminate, and which are not one or zero links removed, are two links removed, etc., according to Ref. 6.

Let C(m) be the probability that a given station is contacted at the m^{th} stage. The probability that a station is contacted for the first time at the m^{th} stage is

$$C(m) \prod_{i=0}^{m-1} [\ 1 - C(i)\ ] \tag{7}$$

Let $B(m) = 1 - C(m)$, Eq. 7 becomes

$$[\ 1 - B(m)\ ] \prod_{i=0}^{m-1} B(i) \tag{7.a}$$

where B(0) is the probability of not selecting a given station at stage zero.

The average number of links emanating from a station that has survived is d. Since each station sends on the average d links, and there are n stations in the communication network, the expected number of links to be traced on the $(m+1)^{th}$ tracing will be

$$X = d\ n\ [\ 1 - B(m)\ ] \prod_{i=0}^{m-1} B(i) \tag{8}$$

The probability that any given station in the aggregate is not contacted by any of these links on the $(m+1)^{th}$ tracing will then be

$$B(m+1) = (\ 1 - 1/n\ )^{X} \tag{9}$$

which, for large n, may be written as

$$B(m+1) = \mathrm{Exp}\{-d\ [\ 1 - B(m)\ ] \prod_{i=0}^{m-1} B(i)\} \tag{10}$$

$$= \mathrm{Exp}\{-d\ [\ \prod_{i=0}^{m-1} B(i) - \prod_{i=0}^{m} B(i)\ ]\}$$

Taking the product of both sides of Eq. 10 with respect to m, yields

$$\prod_{j=1}^{m+1} B(j) = \prod_{j=1}^{m} \mathrm{Exp}\{-d\ [\ \prod_{i=0}^{j-1} B(i) - \prod_{i=0}^{j} B(i)\ ]\} \tag{11}$$

When m goes to infinity, the left hand side of Eq. 11 becomes

$$\lim_{m\to\infty} \prod_{j=1}^{m+1} B(j) = 1 - H \tag{12}$$

The right hand side of Eq. 11 becomes

$$\mathrm{Exp}\{-d \sum_{j=1}^{m} [\ \prod_{i=0}^{j-1} B(i) - \prod_{i=0}^{j} B(i)\ ]\} \tag{13}$$

Inside the braces is

$$-d[B(0) - B(0)\,B(1) + B(0)\,B(1) - \cdots + B(0)\,B(1) \cdots B(m-1)$$
$$- B(0)\,B(1) \cdots B(m)] = -d[B(0) - B(0)\,B(1) \cdots B(m)]$$

Equation 13 becomes

$$\mathrm{Exp}\{-d\ [\ B(0) - \prod_{i=0}^{m} B(i)\ ]\} \tag{13.a}$$

But $B(0)=1-1/n=1$. as n goes to infinity and Eq. 13.a becomes

$$\mathrm{Exp}\{-d\ [\ 1 - \prod_{i=0}^{m} B(i)\ ]\} \qquad (14)$$

Taking the limit of Eq. 14, as m approaches infinity

$$\mathrm{Exp}\{-d\ [\ 1-\ (1\ -\ H)\}]\ =\ \mathrm{Exp}\ (-d\ H) \qquad (15)$$

Equation 12 equals Eq. 15 under these conditions, Thus, $1 - H = \mathrm{Exp}\ (-d\ H)$ or

$$H = 1 - \mathrm{Exp}\ (-d\ \dot{H}) \qquad (16)$$

Substituting Eq. 6 into Eq. 16 yields

$$H = 1 - \mathrm{Exp}\ [\ -s\ P(N)\ H\ ] \qquad (17)$$

Equation 17 is an equation for the probability of survival of the communication network after the enemy attack in terms of the network parameters.

We note that for $H=0$, every d is the solution of the Eq. 16. If $H\neq 0$ then Eq. 16 can be solved explicitly for a given value of d, which is given by

$$d\ =\ \frac{-\ln(1-H)}{H} \qquad (18)$$

The right-hand side of Eq. 18 is analytic in every neighborhood of the origin and tends toward unity as H goes to zero. Expanding that function in power series of H, we have

$$d\ =\ 1 + \frac{H}{2} + \frac{H^2}{3} + \ldots \qquad (19)$$

Negative values of H, being physically meaningless, must be discarded. Thus in the region $0 \leq d \leq 1$, we have $H=0$.

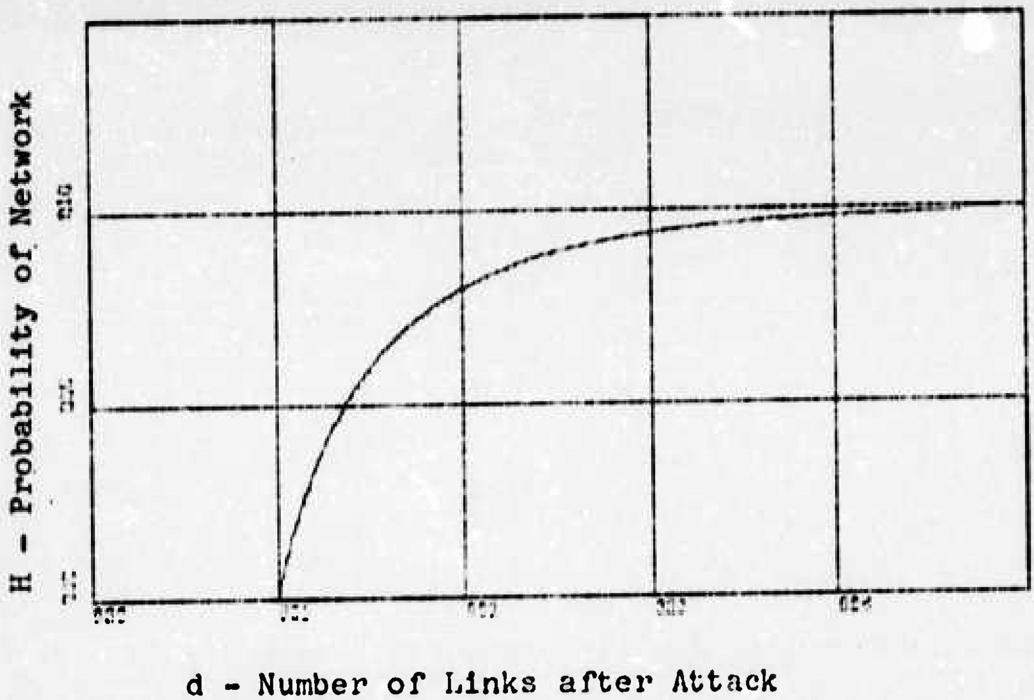


Figure 3

The average number of links d is always an integer. So the value of d equals zero which is physically meaningless.

An examination of the meaningful part of Fig. 3 shows that as long as the number of links does not exceed one link per station H=0, i.e., for a very large n, the number of stations to which there exist paths from an arbitrary station is negligible compared to the total number of stations in the communication network. On the other hand, as the average number of links increases from unity, H increases rather rapidly. For d=2, H reaches about 0.80 of its asymptotic value and is within a fraction of one percent of unity for a quite moderate value of d (say 6).

This means that no matter how large the communication network is, it is nearly certain that there will exist a path

between two stations picked at random, provided only the average number of links is a few times greater than unity.

EXAMPLE 3: Let 24 miles be the average distance between links. Three 5 MT. nuclear weapons are randomly aimed at an area of 3000 square miles. What is the average number of links per station before the enemy attack in order to keep 80 percent survivability of the communication network?

Then, from Fig. 2, for r=12 miles and 5 MT. nuclear weapon

$$p(12) = 0.09$$

The probability of a link being in the damage area, D, is

$$\frac{D}{A} = 0.3025$$

The probability of survival of any given link inside area is

$$P(1) = 0.73$$

Then, the probability of survival, for three 5 MT. nuclear weapons is

$$P(3) = 0.39$$

Then, the average number of links is

$$s = \frac{-\ln(1-H)}{H\ P(3)}$$

$$= 5.16$$

s must be an integer. It is chosen 6.

If the average distance between links is increased to 30 miles, the probability of survival of any given link increases.

So,

$$P(3) = 0.913$$

and the average number of links, s, is 2.5 and chosen 3.

B. THE EFFECT OF THE LENGTH OF THE PATH

Most communication networks have some processing time associated with the links and stations. This processing time may be the time necessary to transmit information through the link or the time needed at station to decode, recode and retransmit the information. In any event, it is usually desirable to limit the time a message remains in the communication network routes. Thus, instead of asking for surviving fraction of stations that can be reached from a given station by a path of no more than m links.

In our urn problem, the drawing of balls are equivalent to sampling a population of n points with replacement; consequently, the same ball may be selected more than once. A more reasonable method of selection is to establish links sequentially. The first station is selected equiprobably out of n possible stations, the second station is selected equiprobably out of the n-1 remaining stations; ... ; the s^{th} station is selected equiprobably out of the n-s remaining stations (none of which has been already selected).

Assume that the links at station v_i are determined by sampling station $v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n$ without replacement a total of d times.

In fact, Eq. 16 is valid for infinite stations. It does not make any difference for large ratios of population-to-sample

size, sampling with and without replacement. If the number of stations is finite, Eq. 16 is not valid. Actually, Eq. 16 is a lower bound for H. This is because a communication network with no parallel links in the same direction has higher probability of being connected [4].

The probability of survival of the communication network can be investigated without replacement for finite stations as follows: Choose an arbitrary station v_1 and let $S_o = v_1$. Let S_1 be the set of stations connected to v_1 by links directed from v_1, ..., and let S_i be the set of stations connected to set of stations S_{i-1} by links directed from S_{i-1}, ... So on.

First, all links emanating from S_o are traced, i.e., the number of stations in S_1 are found, ..., at the i^{th} stage, all links emanating from S_{i-1} which have not already been traced, etc.

We shall rewrite Eq. 7 in terms of another probability E(m), which will be defined as the probability of being contacted for the first time on the m^{th} tracing. Thus,

$$E(m) = [\ 1 - B(m)\ ] \prod_{i=0}^{m-1} B(i) \qquad (7.b)$$

$$= \prod_{i=0}^{m-1} B(i) - \prod_{i=0}^{m} B(i)$$

Taking the sum of both sides of Eq. 7.b with respect to m, yields

$$\sum_{j=0}^{m} E(j) = \sum_{j=0}^{m} \left[\prod_{i=0}^{j-1} B(i) - \prod_{i=0}^{j} B(i) \right]$$

$$= 1 - \prod_{i=0}^{m} B(i) \qquad (20)$$

For $(m+1)^{th}$ stage

$$\sum_{j=0}^{m+1} E(j) = 1 - \prod_{i=0}^{m+1} B(i) \qquad (21)$$

Solving Eq. 20 for $\prod_{i=0}^{m} B(i)$

$$\prod_{i=0}^{m} B(i) = 1 - \sum_{j=0}^{m} E(j) \qquad (20.a)$$

Rewrite Eq. 7.b for $(m+1)^{th}$ stage

$$E(m+1) = [\ 1 - B(m+1)\ ] \prod_{i=0}^{m} B(i) \qquad (7.c)$$

Substituting Eq. 20.a into 7.c

$$E(m+1) = [\ 1 - B(m+1)\ ][\ 1 - \sum_{j=0}^{m} E(j)\ ] \qquad (22)$$

The probability that any given station in the aggregate is not contacted by any of these links on the $(m+1)^{th}$ tracing will be, for large n

$$B(m+1) = \mathrm{Exp}\{-d\ [\ 1 - B(m)\ ] \prod_{i=0}^{m-1} B(i)\} \qquad (10)$$

Since the term of inside the braces of Eq. 10 is equal to $[-d\ E(m)]$, Eq. 10 becomes

$$B(m+1) = \mathrm{Exp}\ [\ -d\ E(m)\ ] \qquad (23)$$

Substituting Eq. 23 into 22 in order to get $E(m+1)$ in terms of $E(m)$. Thus,

$$E(m+1) = [\ 1 - \sum_{j=0}^{m} E(j)\ ]\ \{1 - \mathrm{Exp}\ [\ -d\ E(m)\ ]\} \qquad (24)$$

with $E(0) = 1/n$

$E(m)$ represents the probability that any station is exactly m links removed from a station chosen at random. $E(m)$ is approximately equal to the expected fraction of stations that are connected by at least one path of m links and with fewer than m links to the station picked at random.

When $E(m)$ is known, the probability of survival of the communication network can be figured out. If the only available path between a pair of stations is too long, it may be considered that the enemy has effectively separated the two stations. The probability of survival of the communication network H does not take this factor into account. So,

$$H = \sum_{m=0}^{\infty} E(m) \qquad (25)$$

In fact, if we take the sum of Eq. 24 while m goes infinity, we have

$$H = 1 - \mathrm{Exp}\ (-d\ H)$$

which is the expected result and the same as Eq. 16.

As far as the information transit time is concerned, the determination of probability of survival of the communication network is closely related to the path of length when any given station is connected by a path of length m or less to a station chosen at random. Thus,

$$H(m) = \sum_{k=0}^{m} E(k) \qquad m=1,2,\ldots \tag{26}$$

with $E(0)=1/n$, n is the number of stations.

EXAMPLE 4: A communication network has 100 stations with an average of 20 links per station. Let 27 miles be the average distance between pairs of links. The enemy attacks at some area of 3000 square miles with three 5 MT. nuclear weapons. It is desired to find the probability of survival of the communication network which can be reached, after the enemy attack, from a station chosen at random to any given station by a path of no more than 2 links.

From Fig. 2, for $r=13.5$ miles

$$p(13.5) = 0.50$$

The probability of a link being in the damage area, D, is

$$\frac{D}{A} = 0.3025$$

The probability of survival of a given link inside the area is, for one 5 MT. nuclear weapon

$$P(1) = 0.85$$

For three 5 MT. nuclear weapons

$$P(3) = 0.62$$

Then, from Eq. 16

$$H = 1 - \text{Exp}\ (-20\ 0.62\ H) = 1 - \text{Exp}\ (-12.4\ H)$$

The probability of survival of the communication network H is very close to unity. However, when E(m) is calculated from Eq. 24

$$E(0) = 1/100 = 0.01$$

$$E(1) = (1 - 0.01)\ [\ 1 - \text{Exp}\ (-12.4\ 0.01)\ ] = 0.115$$

$$E(2) = (1 - 0.125)[\ 1 - \text{Exp}\ (-12.4\ 0.115)] = 0.665$$

Therefore, although nearly 100 percent of the stations can be reached from a station chosen at random, $H(3) = 0.01 + 0.115 + 0.665 = 0.79$ and only 79 percent of the stations can be reached with paths of two or less links.

IV. FINITE NETWORK

A. COMPUTATION OF SURVIVABILITY BY EXACT METHOD

A communication network has n stations and m links. Each link has a finite probability of survival; they are denoted by p_1, p_2, ..., p_m under the states y_1, y_2, ..., y_m, respectively. It is assumed that each link is associated with a statistically independent random variable with only two possible states, namely, the state in which the link is in operation and the state in which it is not in operation.

If the link b_i exists in the network with the probability of survival p_i, this means that y_i=1. Let the state Y_k of the entire network be described by a state vector (y_1, y_2, ..., y_m) where y_i=1 or 0 according to whether the link b_i is in an operating state or not. The totality of all the 2^n state vectors forms the sample space, and each state vector corresponds to a vertex of o unit m-dimensional cube. Since the links b_1, b_2, ..., b_m are considered statistically independent, the probability of survival that the state Y_k exists [5].

$$P_k = \prod_{i=0}^{m} p_i^{y_i} (1 - p_i)^{1-y_i} \qquad (27)$$

where p_i is the probability of survival of the link b_i under state y_i; and P_k is the probability of the state k.

In this communication network, all links are assumed identical with equal probability of survival. Then Eq. 27 can be

written as follows:

$$P_k = \prod_{i=0}^{m} p^{y_i} (1 - p)^{1-y_i} \qquad (27.a)$$

A path between two stations, say, between the station v_1 and v_j, is a subset of the links of the communication network graph of the form $(\overline{v_1 v_2}, \overline{v_2 v_3}, \ldots, \overline{v_{j-1} v_j})$. A "loop" in a graph is a path with one additional link joining the two stations of the path. A "tree" is an n-station communication network graph is a set of n-1 links that contains a path between every pair of station in the graph. It can easily be shown that any set of n-1 links that contains no loop is a tree.

In order to maintain communication among all stations in a network, at least one path between any two stations of the network is needed. It is well known that a finite graph is a tree if and only if there exists exactly one path between two stations. Therefore, the communication is assured if and only if there exists at least a tree in the network.

The probability of survival of a communication network is defined as the probability of the communication between every pair of its stations. So, it is the algebraic sum of the probabilities of all the possible states which contains at least a tree of the network. Therefore, [5]

$$H = \sum_{k=1}^{2m} t_k P_k \qquad (28)$$

where P_k is the probability of the state k, t_k is 1 or 0 if the state k contains a tree or no tree in the network as its subnetwork respectively.

EXAMPLE 5: What is the probability of survival of the communication network which is shown in Fig. 4?

From Eqs. 27 and 28

$$
\begin{aligned}
H = {} & p_1p_2q_3q_4q_5p_6 + p_1p_2q_3q_4p_5q_6 + p_1p_2q_3p_4q_5q_6 + p_1q_2p_3p_4q_5q_6 + \\
& p_1q_2p_3q_4p_5q_6 + p_1q_2p_3q_4q_5p_6 + p_1q_2q_3p_4q_5p_6 + p_1q_2q_3q_4p_5p_6 + \\
& p_1q_2q_3p_4q_5p_6 + q_1p_2p_3p_4q_5q_6 + q_1p_2p_3q_4p_5q_6 + q_1p_2q_3p_4q_5p_6 + \\
& q_1p_2q_3q_4p_5p_6 + q_1p_2p_3q_4q_5p_6 + p_1p_2p_3q_4q_5p_6 + p_1p_2p_3q_4p_5q_6 + \\
& p_1p_2q_3q_4p_5p_6 + p_1p_2q_3p_4q_5p_6 + p_1p_2q_3p_4p_5q_6 + p_1q_2p_3p_4q_5p_6 + \\
& p_1q_2q_3p_4p_5p_6 + p_1q_2p_3q_4p_5p_6 + q_1p_2p_3p_4p_5q_6 + q_1p_2p_3q_4p_5p_6 + \\
& q_1p_2p_3p_4q_5p_6 + p_1p_2p_3p_4q_5p_6 + p_1p_2p_3p_4p_5q_6 + p_1p_2p_3q_4p_5p_6 + \\
& p_1q_2p_3p_4p_5p_6 + p_1p_2q_3p_4p_5p_6 + q_1p_2p_3p_4p_5p_6 + p_1p_2p_3p_4p_5p_6
\end{aligned}
$$

where p_i is the probability of link b_i, and $q_i = 1 - p_i$, $i = 1, \ldots, 6$.

If $p_1 = \ldots = p_6$, above equation becomes

$$H = 13\ p^3\ q^3 + 13\ p^4\ q^2 + 6\ p^5\ q + p^6$$

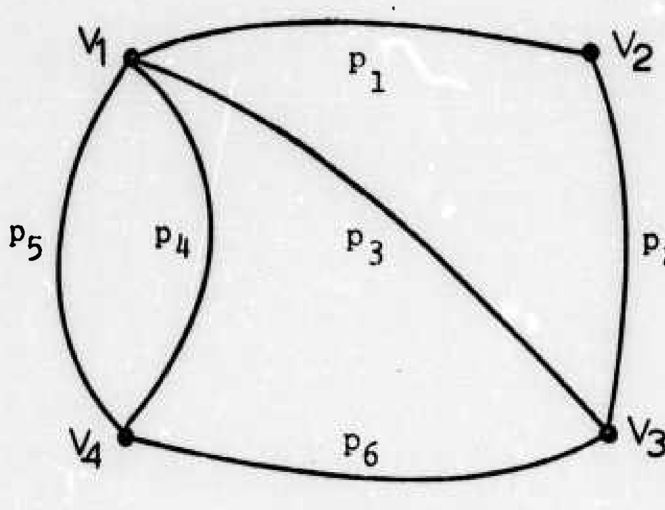


Figure 4

B. COMPUTATION OF SURVIVABILITY BY APPROXIMATION METHOD

In the exact method of calculation of the probability of the communication network, first, all the trees in the network must be found. Secondly, P_k that is the probability of the state k has to be computed for all possible states. So, if the network has a large number of stations and links, computation takes a long time.

In this section, a finite network whose links have the same finite probability of survival is considered. A general method is given to compute the approximate probability of survival of the communication network. An equivalent network of the communication network can be used to compute the approximate probability of survival. If each link is assigned a unit capacity, the maximum flow between any pair of stations is equal to the corresponding mim-cutset, which must be removed in order to separate these stations.

1. Equivalent Network

The network flow problem was first considered by Ford and Fulkerson [10] who introduced the basic concepts of flow, cut, etc., and provided the main tool, the maximum-flow minimum-cut theorem. Ford and Fulkerson discussed the flow between two special points, the source and the sink. Gomory and Hu [9] studied the problem of multi-terminal flow and suggested the use of the equivalent network which has the same flow of the original network.

The construction of Gomory and Hu is described as follows: Select two nodes arbitrarily and solve a maximal

flow problem between them. This locates a minimal cut $(X,\bar{X})$, which we represent symbolically by two nodes connected by a link of capacity e_1, as in Fig. 5.

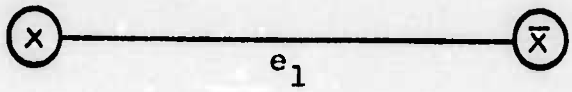


Figure 5

In one node, the individual nodes of X are listed; in the other, those of $\bar{X}$. Next, choose two nodes in X, and solve the resulting maximal flow problem in the $\bar{X}$-condensed network, i.e., all the nodes of $\bar{X}$ can be shown as a single node. The resulting minimal cut has capacity e_2 and is represented by a link of this capacity connecting the two parts into which X is divided by the cut, say X_1 and X_2. The node $\bar{X}$ is connected to X_1 if it is in the same part of the cut as X_1; to X_2 otherwise, as in Fig. 6.

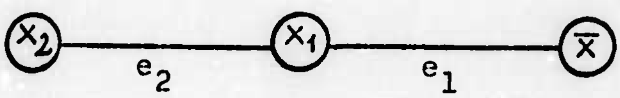


Figure 6

This process discussed above is continued, and at each stage of the construction some set Y, consisting of more than one node, is chosen from the tree diagram at that stage. The set Y will have a certain number of links connected to it in this tree. All of the sets that can be reached from Y by paths using one of these links are condensed into a single

node for the next maximal flow problem. This is done for each link connected to Y in the tree. In the resulting network a maximal flow problem is solved between two nodes of Y. The set Y is partitioned into Y_1 and Y_2 by the minimal cut thus found; this is represented in the new tree by a link having capacity equal to the cut capacity joining Y_1 and Y_2; the other nodes of the old tree are connected to Y_1 if they are in the Y_1 part of the cut; to Y_2 otherwise.

To illustrate the general step of the construction, suppose that we have arrived at the tree diagram of Fig. 7, with Y to be split. Removal of the links connected to Y leaves the connected components Y; X_1; X_2, X_3; X_4, X_5, X_6. Then in the original network the nodes X_1 are condensed, as are those $X_2 \cup X_3$, and $X_4 \cup X_5 \cup X_6$.

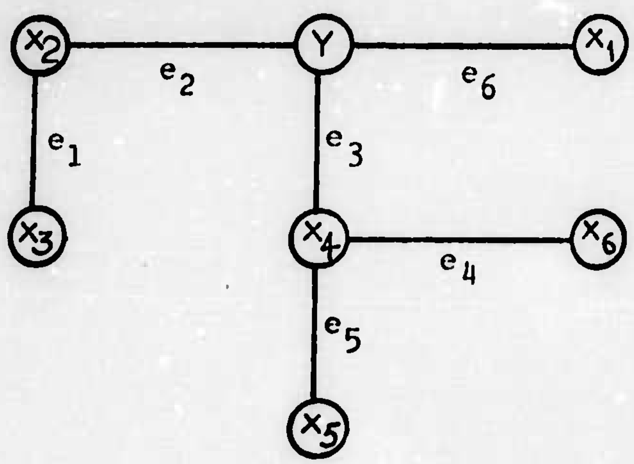


Figure 7

Solving a maximal flow problem between two nodes of Y in the condensed network might then lead to the new tree, as shown in Fig. 8.

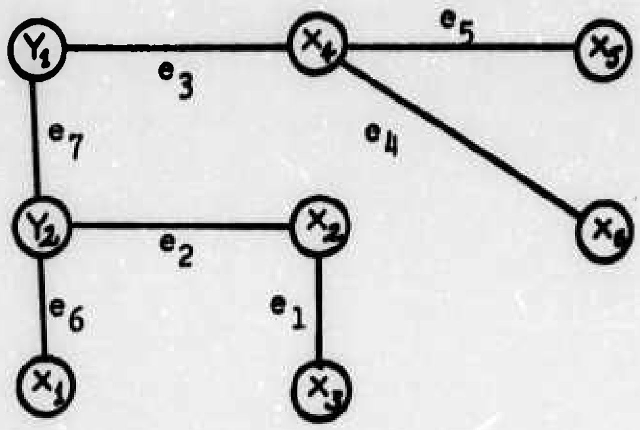


Figure 8

The process is repeated until all the sets consist of one node each. If the original network has n nodes, this point is reached after n-1 maximal flow problems have been solved, since the final diagram is a tree on n nodes, each link of which has been created by solving a flow problem. The number e_k attached to the k^{th} link of the final tree is the capacity of this link.

EXAMPLE 6:

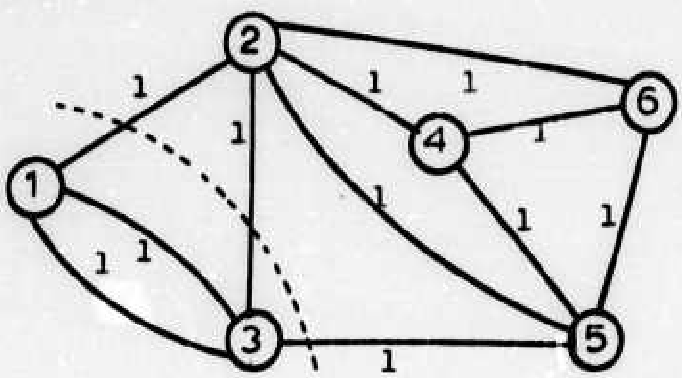


Figure 9

To begin the analysis for the network of Fig. 9, arbitrarily select node 1 and 6 for the first flow problem. This yields the cutset ({1,3} , {2,4,5,6}) represented by the tree of Fig. 10.

Figure 10

Taking 1 and 3 for the next flow problem and condensing 2,4,5,6 gives the network of Fig. 11, with the subsequent outset

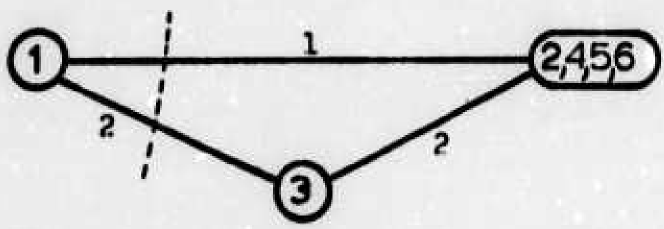


Figure 11

({1} , {3,4,5,6,2}). Hence the tree of Fig. 10 becomes

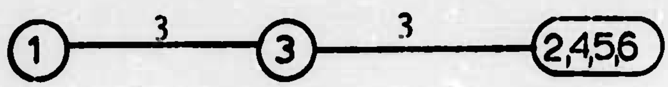


Figure 12

Next choose 2 and 4 the condensed network is shown in Fig. 13 with the cutset ({4} , {1,2,3,5,6})

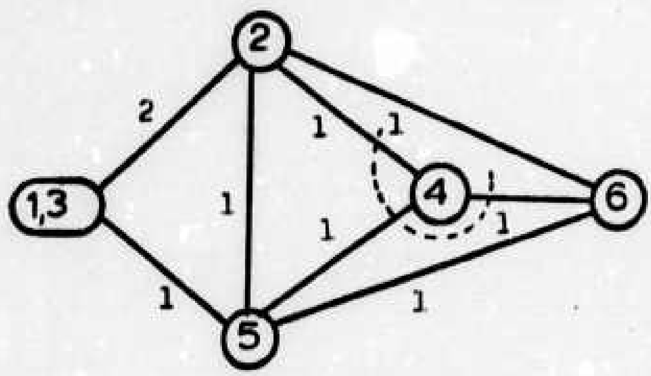


Figure 13

Hence the tree of Fig. 12 becomes

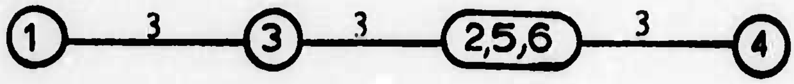


Figure 14

Selecting 2 and 5 for the next flow problem and condensing yields Fig. 15 with the cutset ({5} , {1,2,3,4,6})

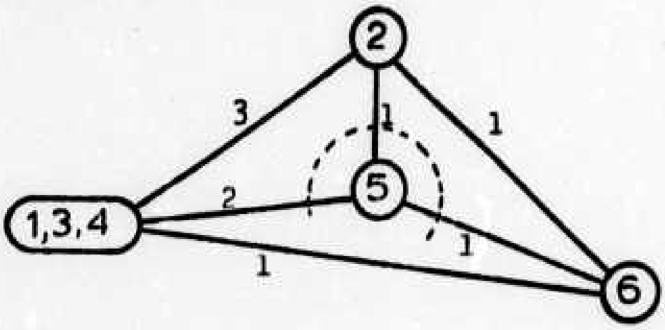


Figure 15

Thus the tree diagram at this stage is as shown in Fig. 16

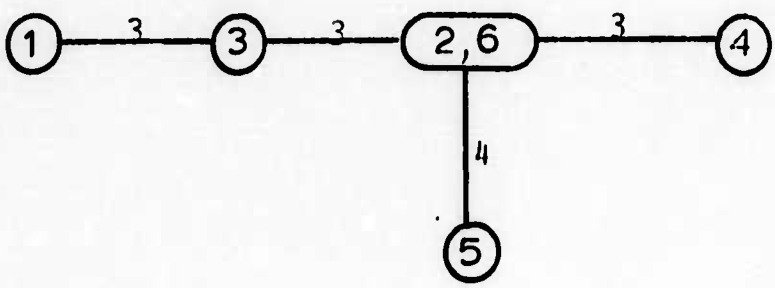


Figure 16

Finally choose 2 and 6 to get the condensed network of Fig. 17 with the cutset ({1,2,3,4,5} , {6})

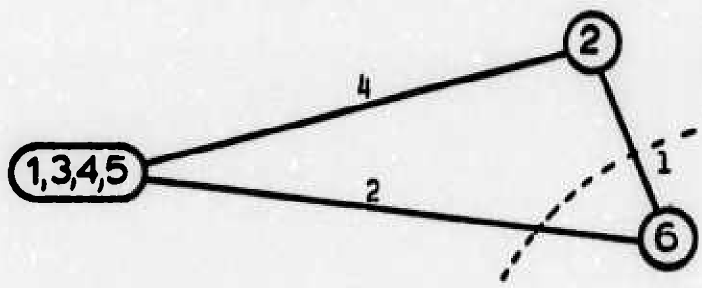


Figure 17

Consequently the final cut-tree is as shown in Fig. 18

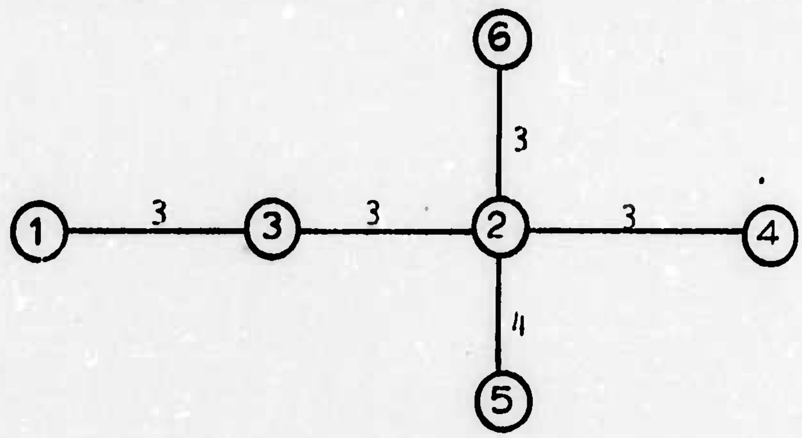


Figure 18

2. Computation of Approximated Value of Survivability

Let an equivalent network be a tree with n nodes and n-1 links. Each link has a capacity denoted by e_i, where $i=1,2,...,n-1$. The tree links with capacity e_i can be represented by e_i parallel links with unit capacity. As discussed in the previous section, e_i is always an integer. So, the network of Fig. 8 can be redrawn as in Fig. 19, in which the total number of links between Y_2 and X_1 is e_6, ..., X_4 and X_6 is equal to e_4.

The probability of survival between two nodes, say, X_1 and Y_2 can be calculated. There are e_6 links which have

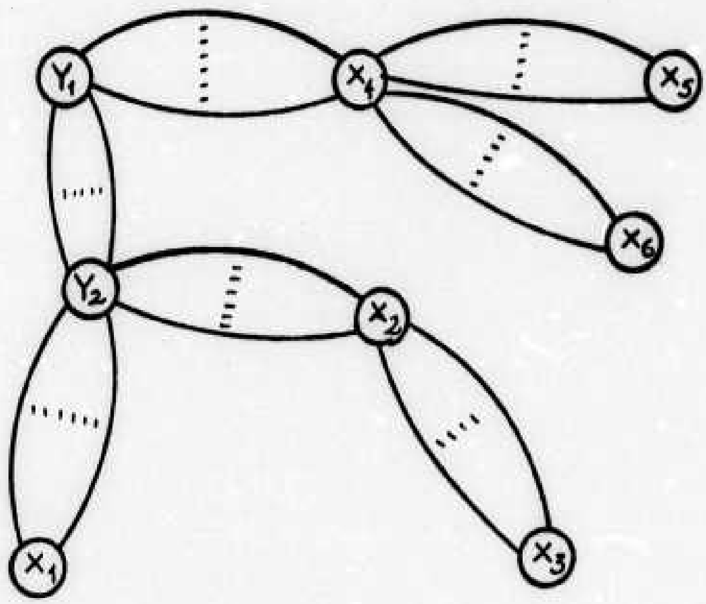


Figure 19

the same probability of survival. Then,

$$g(6) = 1 - (1-p)\,(1-p)\,\ldots\,(1-p)$$

$$= 1 - q^{e_6} \qquad (29)$$

where g(i) is the probability of survival between two nodes with link capacity e_i.

In general,

$$g(i) = 1 - q^{e_i} \qquad (29.a)$$

where q=1-p.

The network of Fig. 19 could be redrawn as in Fig. 20 so that each link has probability g(i), where i=1,2,...,n-1.

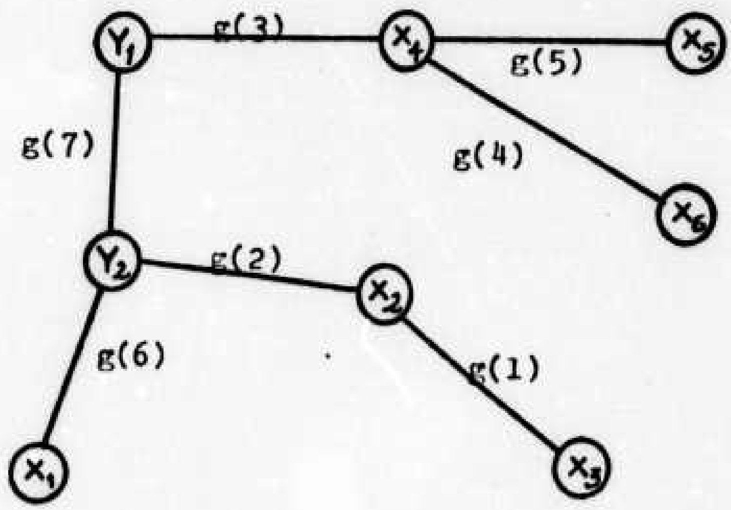


Figure 20

The probability of survival of the network of Fig. 20 can easily be computed as follows:

$$H = g(1)\ g(2)\ g(3)\ g(4)\ g(5)\ g(6)\ g(7)$$

For n-node network

$$H = g(1)\ g(2)\ \ldots\ g(i)\ \ldots\ g(n-1)$$

$$= \prod_{j=1}^{n-1} g(j) \tag{30}$$

Substituting Eq. 29.a into 30, we have

$$H = \prod_{j=1}^{n-1} \left(1 - q^{e_j} \right) \tag{31}$$

The exact and approximation probability of six different finite networks are computed for different probability of survival of the links using Eqs. 28 and 31, respectively. They are shown in Appendix A. The tables for these computations are shown in Appendix B.

All of the six figures have some similar characteristics. The approximated value is always greater than the exact value for any value of p. Equation 31 is a reasonable approximation for computing the probability of survival of the network, because the approximation is about equal to the exact value when $p \leq 0.20$ and $p \geq 0.80$. The average maximum error is only 3.2 percent when p is in this region. The average maximum error for these networks occurs at $p = 0.55$, and it is equal to 8 percent, as shown in Appendix B.

Using modified equivalent network, the probability of survival of the network can be calculated as before. The modified equivalent of the communication network might be obtained as follows: Follow the same procedure used in getting the equivalent network. If the original network has n nodes, solve n-1 maximal flow problems, since the final diagram is a tree with n nodes. The k^{th} link has the capacity e_k. Draw the tree that links with capacity e_k between nodes X_k and X_{k+1}. Represent the tree links by e_k parallel links with unity capacities. Remove one link between nodes X_i and X_{i+1}, where e_i is maximum in e_k $k=1,2,\cdots,n-1$. The final diagram is the modified equivalent network.

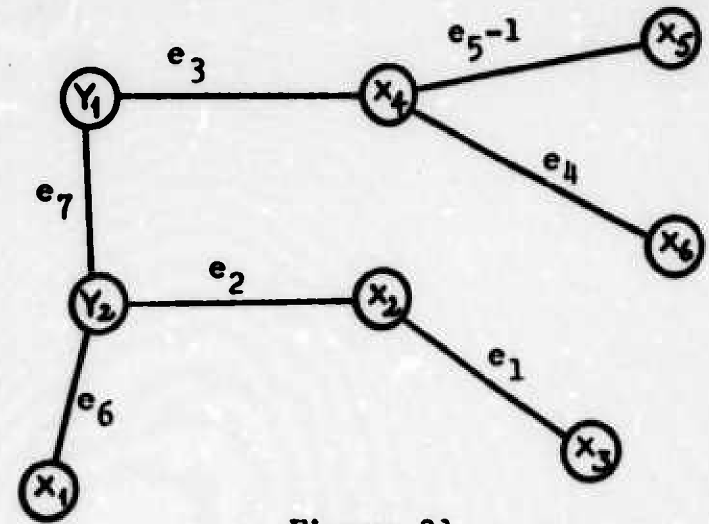


Figure 21

Assume e_5 is the maximum integer in e_k, $k=1,2,\cdots,6$. The modified equivalent of Fig. 8 can be drawn as in Fig. 21. The survivability of Fig. 21 is

$$H = g(1)\ g(2)\ g(3)\ g(4)\ g'(5)\ g(6)\ g(7)$$

where $g'(5) = 1 - q^{e_5 - 1}$

For n-node network

$$H = \frac{1-q^{e_1-1}}{1-q^{e_1}} \prod_{k=1}^{n-1} (1 - q^{e_k}) \qquad (32)$$

where e_1 is the biggest integer in e_k, $k=1,2,\cdots,n-1$.

Equation 32 gives better approximation for computation of the probability of survival of the network. It can easily be seen in Appendix B, the average maximum error is only 3.2 percent instead of 8 percent. Also for small p, say $p \leq 0.50$, the exact and approximated value are almost the same.

V. CONCLUSION

If the number of stations in a random communication network is extremely large, then the first method (Eq. 16) is best of the four methods for computing the probability of survival. In Eq. 16, the path of length is not considered important. It gives a lower bound for the probability of survival of the communication networks.

When the path of length is a major factor, the second method (Eq. 26) results in a better computation of survivability.

The third and fourth methods (Eqs. 31 and 32, respectively) are approximations of the survivability of finite networks. The third method is based on the equivalent network which utilizes the min-cut maximum-flow theorem and has been applied to the computation of survivability of six networks. The results are reasonable, i.e., the average maximum error between the exact method and this approximation is 8 percent. The fourth method is based on the modified equivalent network which also utilizes the min-cut maximum-flow theorem. Again this method is applied to the same six networks, however, the results are significantly improved.

Some suggestions for further studies are given below.

For random networks:

1. Systems with nonuniform links and distance bias,
2. Systems with repair and memory.

For finite network: Methods three and four are applied only to six simple networks.

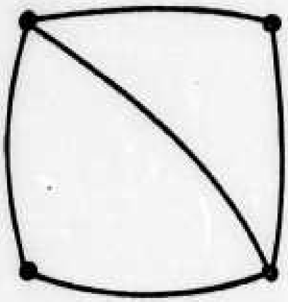
1. They may be applied to more complex networks in order to compare which method is best approximation.

The computation of survivability is based on the identical probability of survival of links.

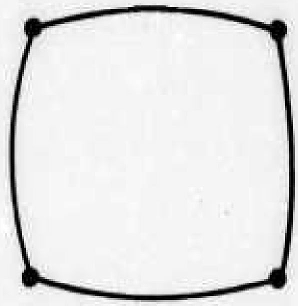
2. They may be extended for unequal probability of survival of links.

APPENDIX A

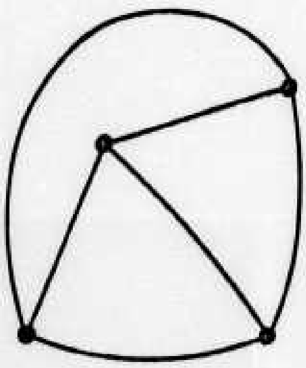
SIMULATION OF THE PROBABILITY OF SURVIVAL OF NETWORKS



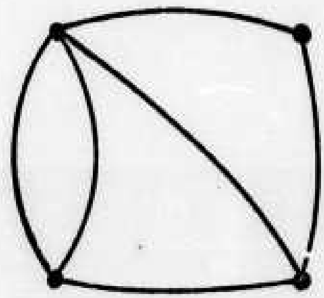
Network-One



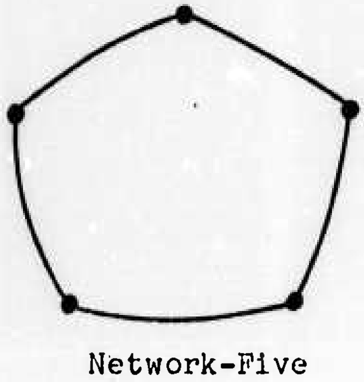
Network-Two



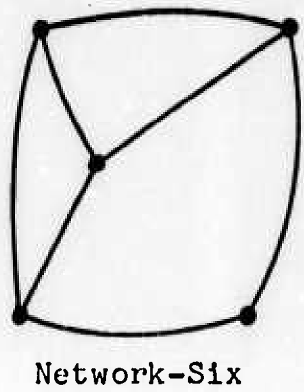
Network-Three



Network-Four



Network-Five



Network-Six

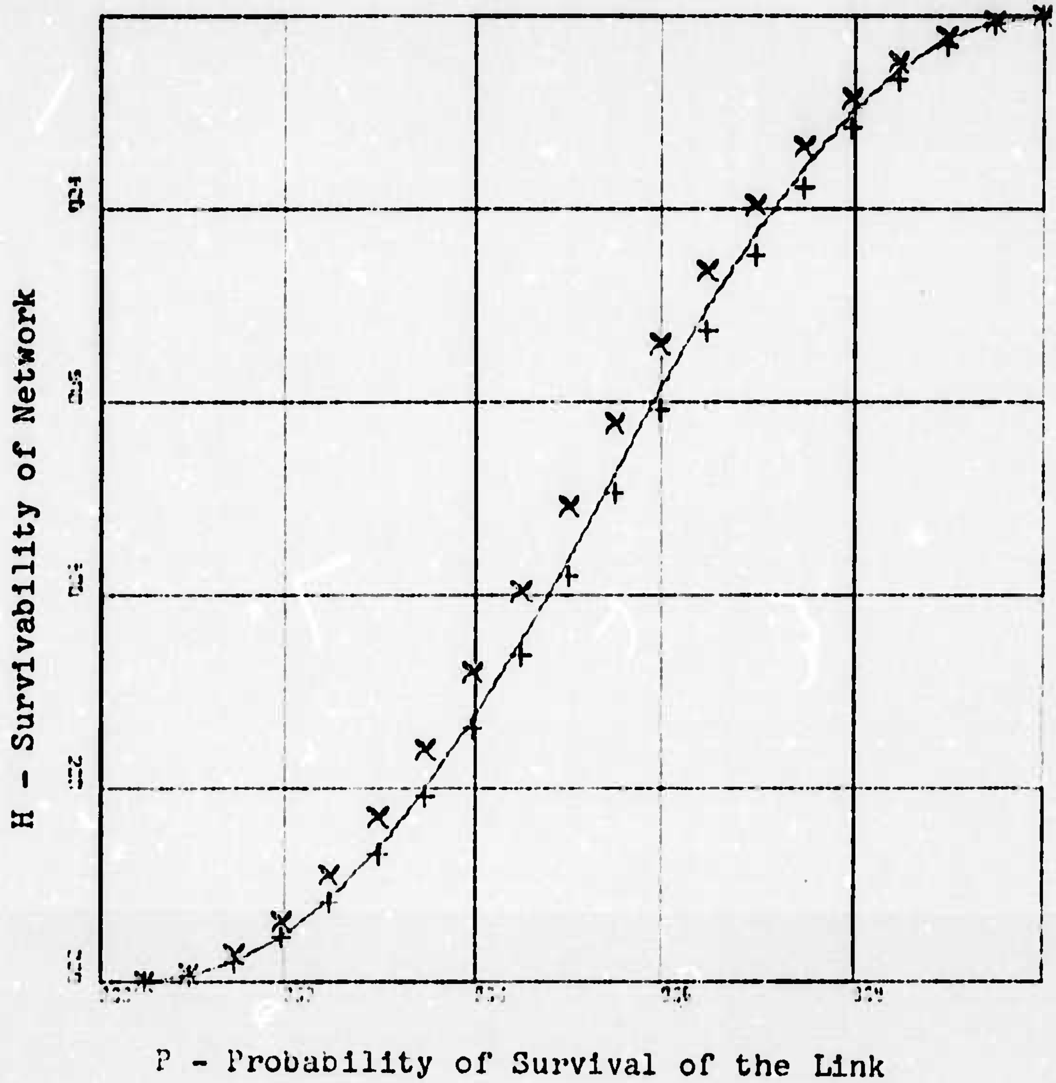


Figure A-1. Simulation of the Probability of Survival of Network One.

Solid line represents the exact value, cross and plus sign represent approximated value using Eqs. 31 and 32, respectively.

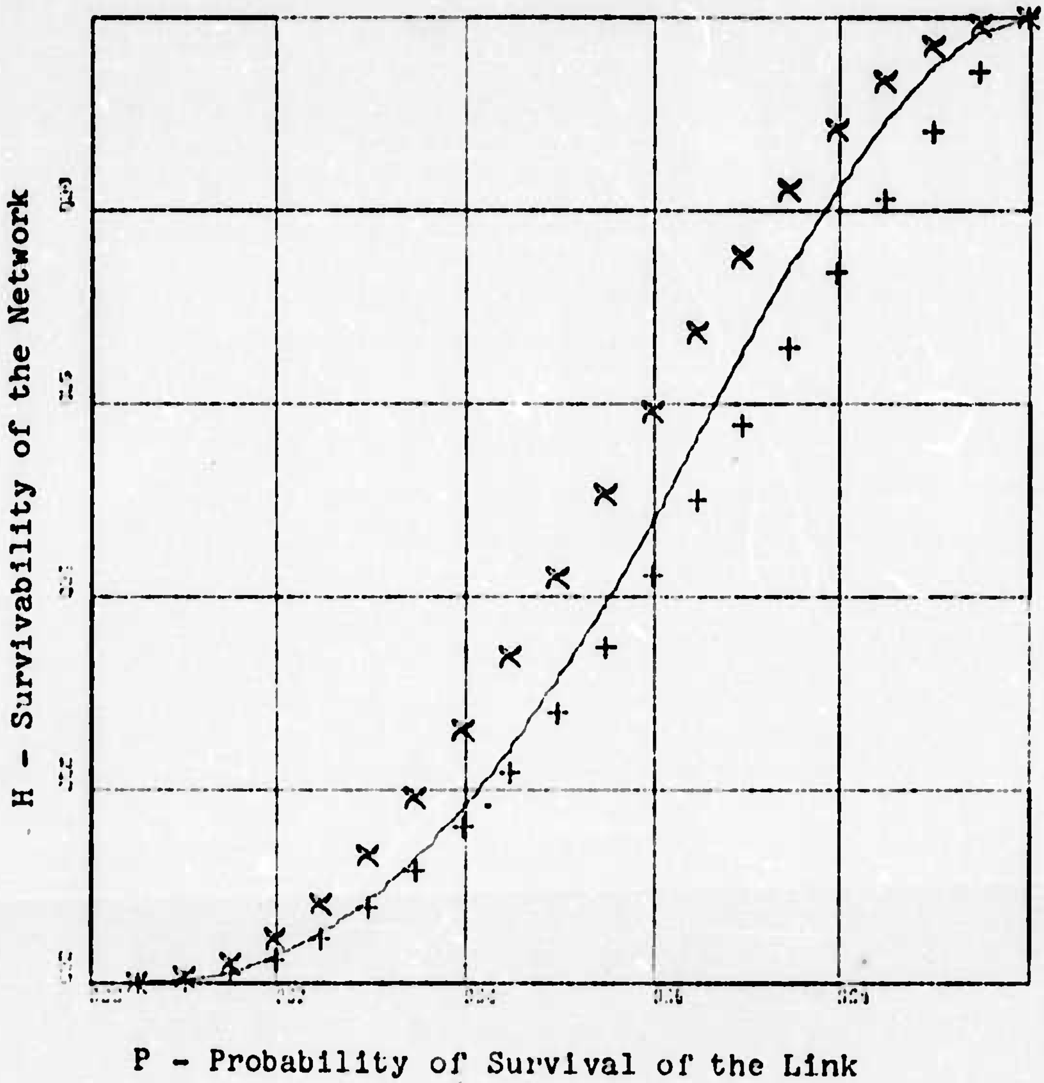


Figure A-2. Simulation of the Probability of Survival of Network Two.

Solid line represents the exact value, cross and plus sign represent approximated value using Eqs. 31 and 32, respectively.

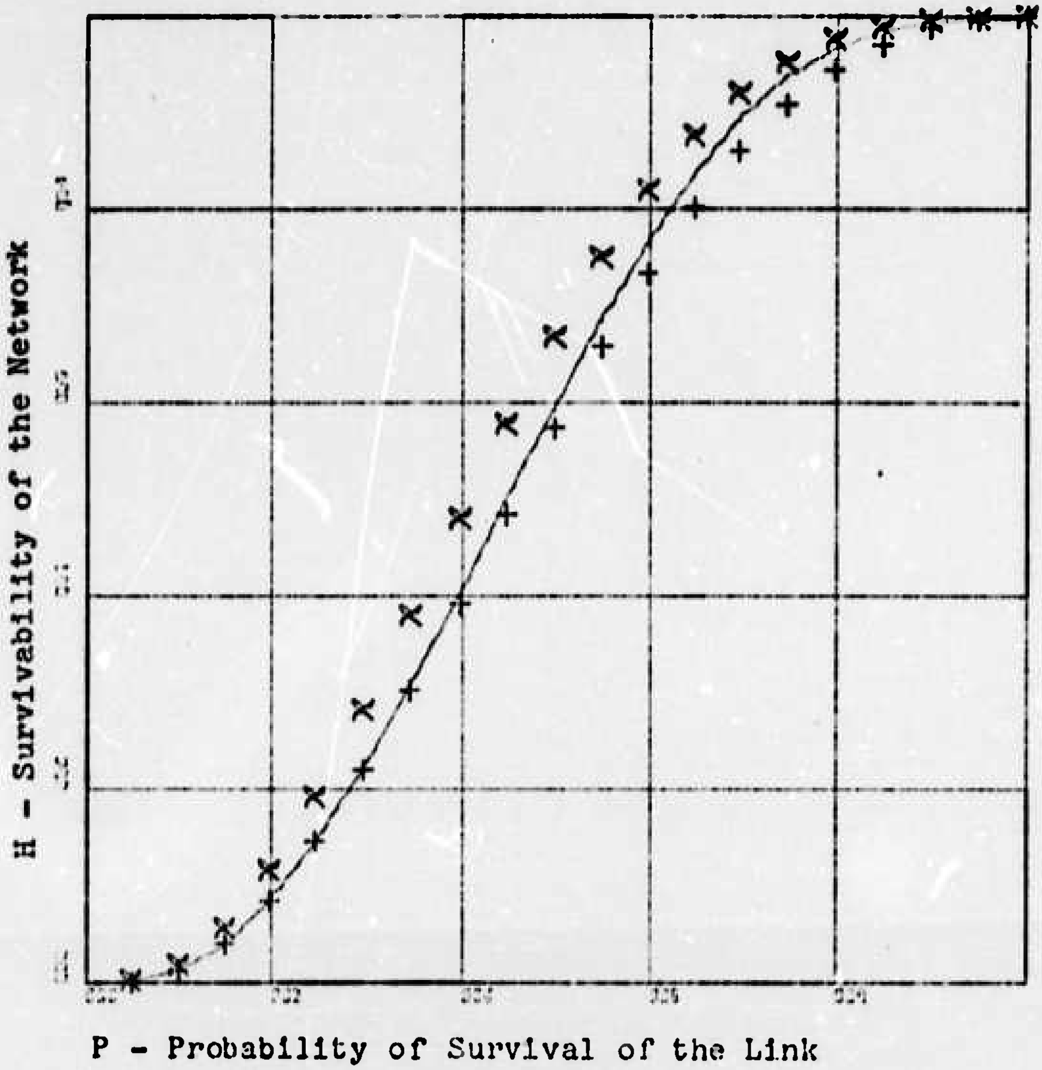
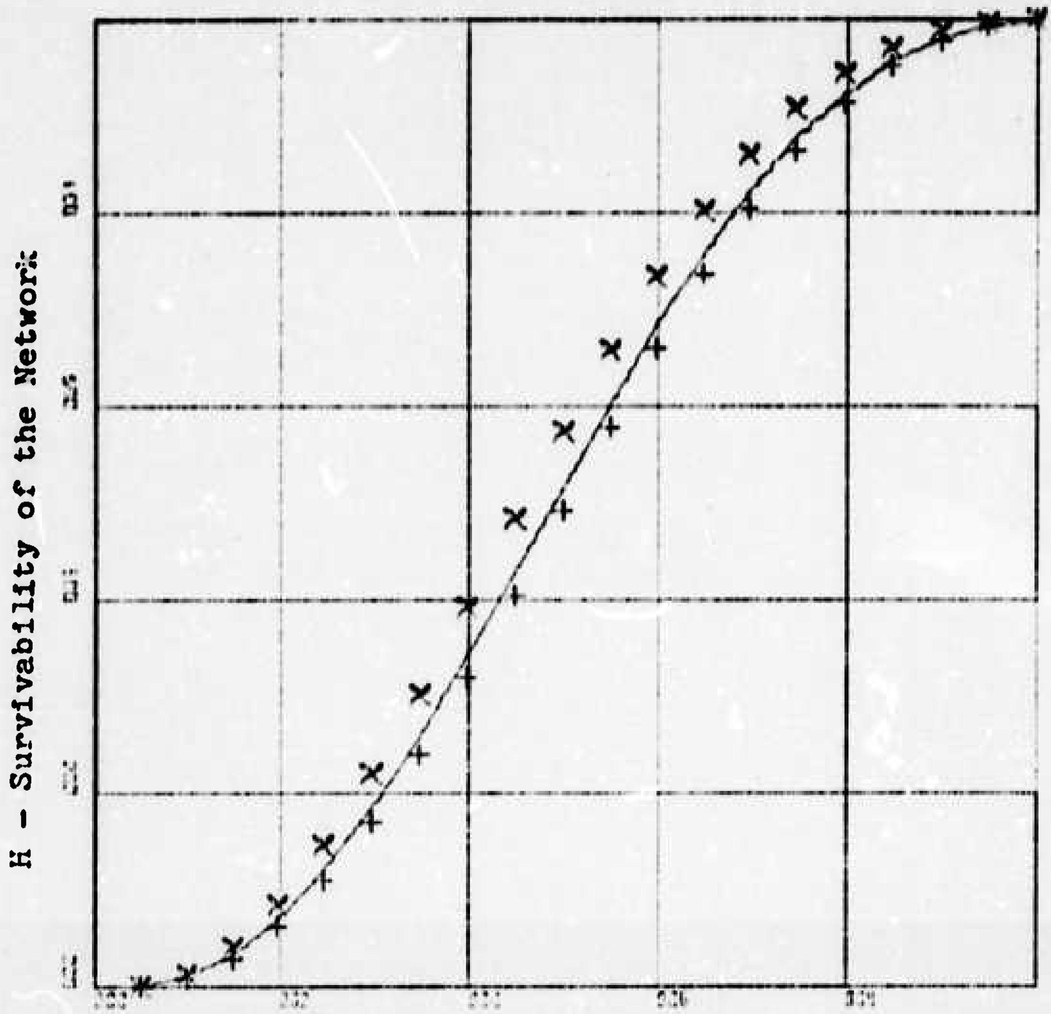


Figure A-3. Simulation of the Probability of Survival of Network Three.

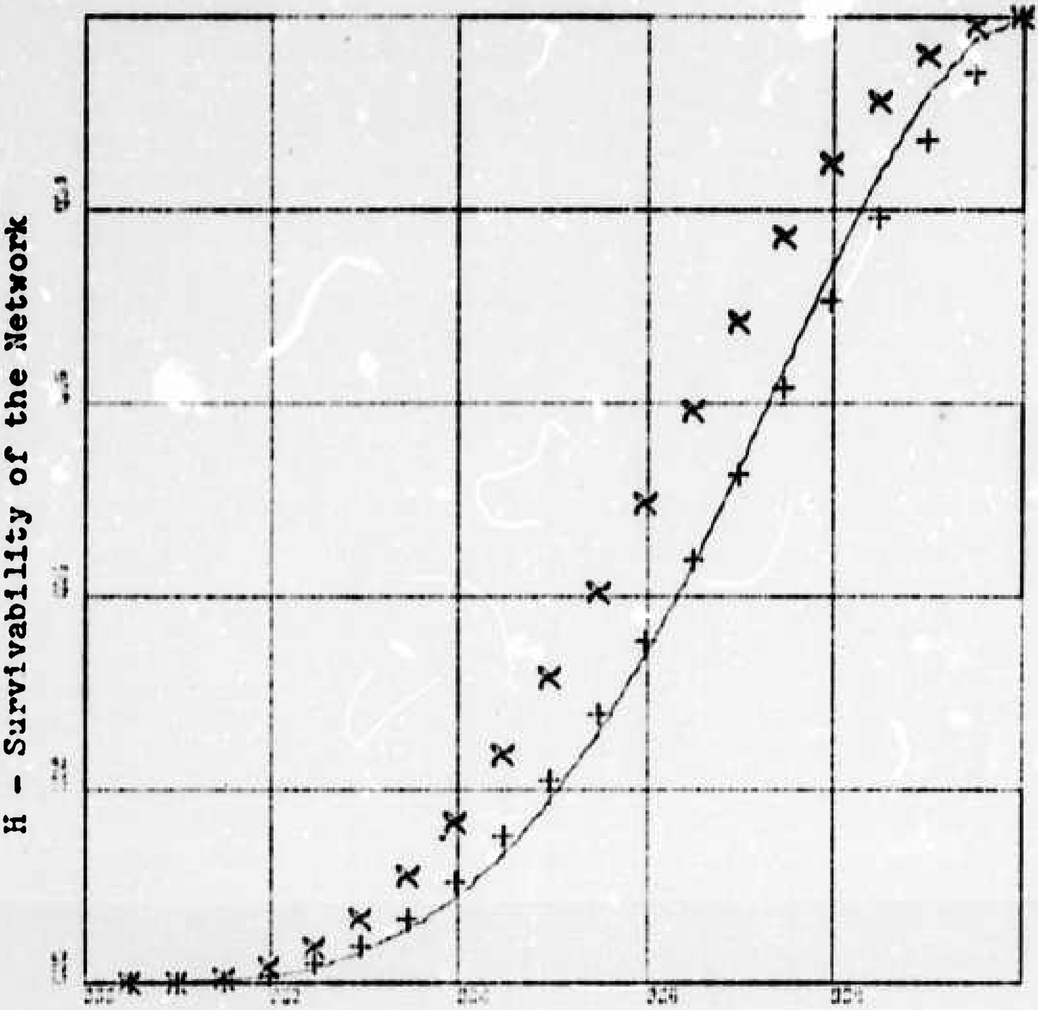
Solid line represents the exact value, cross and plus sign represent approximated value using Eqs. 31 and 32, respectively.



P - Probability of Survival of the Link

Figure A-4. Simulation of the Probability of Survival of Network Four.

Solid line represents the exact value, cross and plus sign represent approximated value using Eqs. 31 and 32, respectively.



P - Probability of Survival of the Link

Figure A-5. Simulation of the Probability of Survival of Network Five.

Solid line represents the exact value, cross and plus sign represent approximated value using Eqs. 31 and 32, respectively.

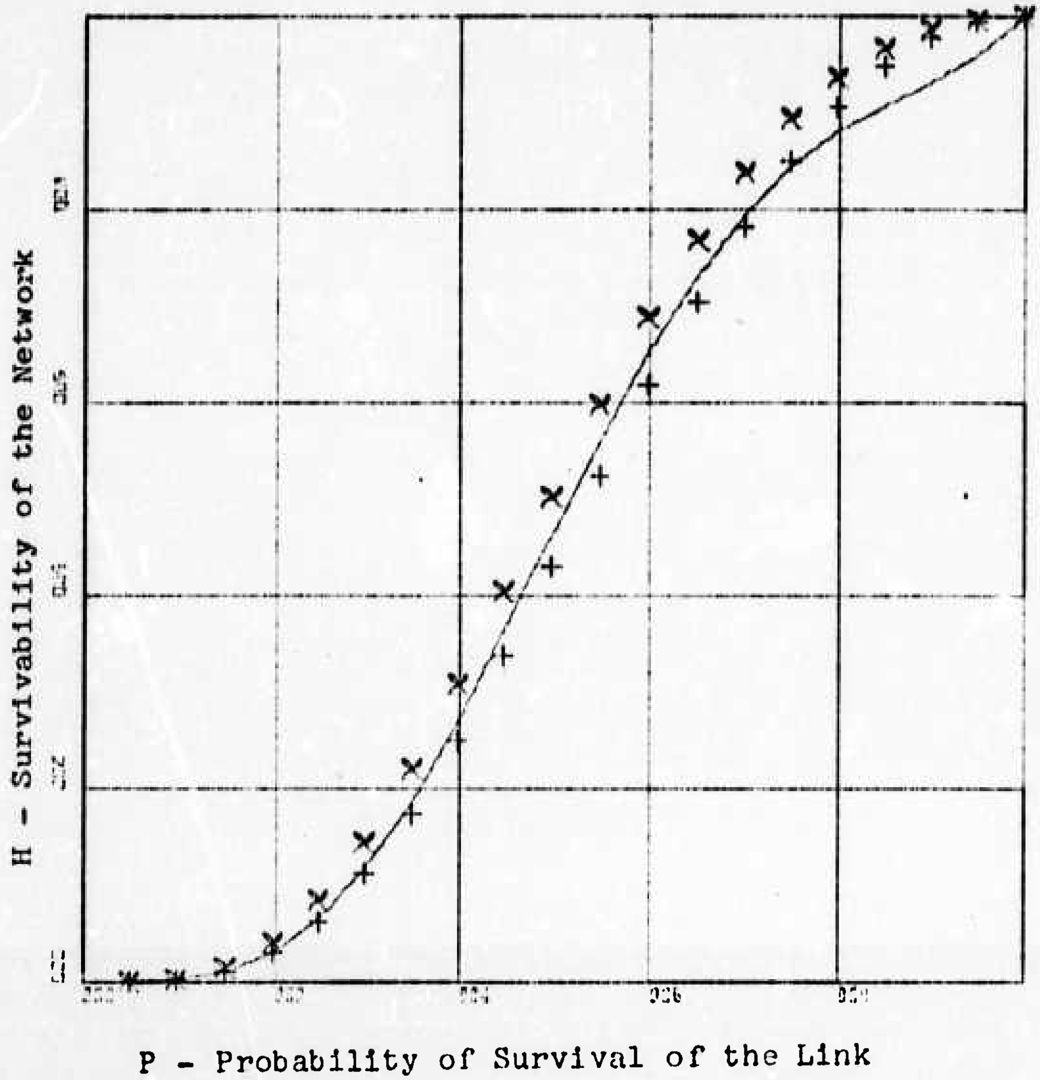
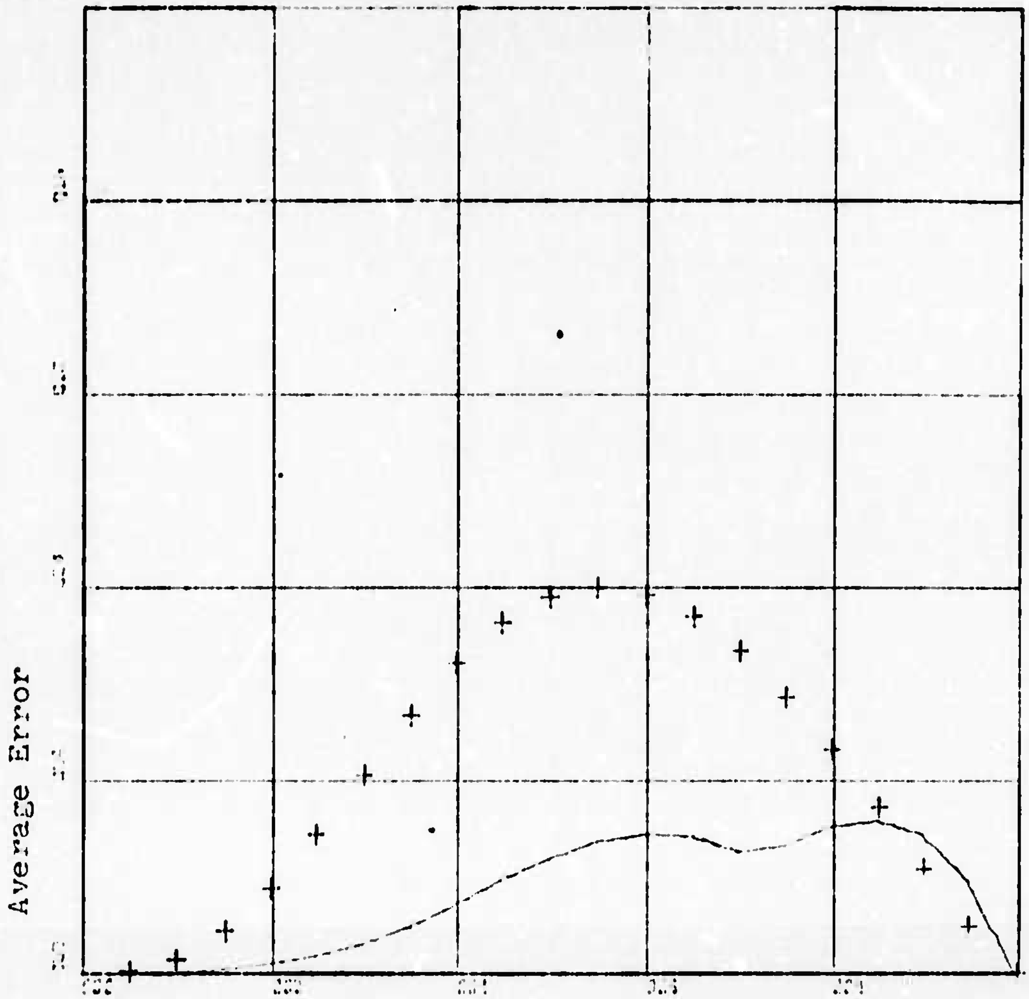


Figure A-6. Simulation of the Probability of Survival of Network Six.

Solid line represents the exact value, cross and plus sign represent approximated value using Eqs. 31 and 32, respectively.



P - Probability of Survival of the Link

Figure B-1

Simulation of the average errors between the exact method and approximations methods.

Solid line and plus sign represent the errors of Eqs. 32 and 31, respectively.

APPENDIX B

TABLES OF COMPUTATION OF SURVIVABILITY

Table B - 1

Link Prob.	Network One			Network Two		
	Eq. 28	Eq. 31	Eq. 32	Eq. 28	Eq. 31	Eq. 32
0.05	0.001	0.001	0.001	0.000	0.001	0.000
0.10	0.007	0.010	0.007	0.004	0.007	0.004
0.15	0.022	0.030	0.021	0.012	0.021	0.012
0.20	0.048	0.063	0.047	0.027	0.047	0.026
0.25	0.086	0.111	0.084	0.051	0.084	0.048
0.30	0.137	0.171	0.133	0.084	0.133	0.078
0.35	0.199	0.242	0.193	0.126	0.193	0.117
0.40	0.271	0.321	0.262	0.179	0.262	0.164
0.45	0.352	0.406	0.339	0.241	0.339	0.219
0.50	0.437	0.492	0.422	0.312	0.422	0.281
0.55	0.526	0.578	0.507	0.391	0.507	0.350
0.60	0.613	0.660	0.593	0.475	0.593	0.423
0.65	0.698	0.737	0.676	0.563	0.676	0.501
0.70	0.775	0.806	0.754	0.652	0.754	0.580
0.75	0.844	0.865	0.824	0.738	0.824	0.659
0.80	0.901	0.914	0.885	0.819	0.885	0.737
0.85	0.946	0.952	0.934	0.890	0.934	0.812
0.90	0.977	0.979	0.970	0.948	0.970	0.882
0.95	0.995	0.995	0.993	0.986	0.993	0.945
1.00	1.000	1.000	1.000	1.000	1.000	1.000

Table B - 2

Link Prob.	Network Three Eq. 28	Eq. 31	Eq. 32	Network Four Eq. 28	Eq. 31	Eq. 32
0.05	0.002	0.003	0.002	0.001	0.002	0.001
0.10	0.013	0.020	0.014	0.011	0.014	0.010
0.15	0.039	0.057	0.041	0.032	0.041	0.030
0.20	0.082	0.116	0.086	0.068	0.086	0.063
0.25	0.143	0.193	0.146	0.119	0.146	0.111
0.30	0.219	0.284	0.220	0.183	0.220	0.171
0.35	0.306	0.382	0.304	0.258	0.304	0.242
0.40	0.400	0.482	0.393	0.340	0.393	0.321
0.45	0.498	0.579	0.485	0.428	0.485	0.406
0.50	0.594	0.670	0.574	0.516	0.574	0.492
0.55	0.684	0.751	0.659	0.602	0.659	0.578
0.60	0.766	0.820	0.736	0.683	0.736	0.660
0.65	0.835	0.877	0.804	0.756	0.804	0.737
0.70	0.892	0.921	0.862	0.821	0.862	0.806
0.75	0.936	0.954	0.908	0.877	0.980	0.865
0.80	0.967	0.976	0.945	0.922	0.945	0.914
0.85	0.986	0.990	0.971	0.956	0.971	0.952
0.90	0.996	0.997	0.988	0.981	0.988	0.979
0.95	0.999	1.000	0.997	0.995	0.997	0.995
1.00	1.000	1.000	1.000	1.000	1.000	1.000

Table B - 3

Link Prob.	Network Five Eq. 28	Network Five Eq. 31	Network Five Eq. 32	Network Six Eq. 28	Network Six Eq. 31	Network Six Eq. 32
0.05	0.000	0.000	0.000	0.000	0.000	0.000
0.10	0.000	0.001	0.001	0.003	0.004	0.003
0.15	0.002	0.006	0.003	0.011	0.016	0.011
0.20	0.007	0.017	0.009	0.031	0.042	0.031
0.25	0.016	0.037	0.021	0.066	0.085	0.064
0.30	0.031	0.068	0.040	0.118	0.145	0.112
0.35	0.054	0.111	0.067	0.186	0.220	0.175
0.40	0.087	0.168	0.105	0.269	0.308	0.252
0.45	0.131	0.237	0.153	0.362	0.404	0.338
0.50	0.187	0.316	0.211	0.461	0.502	0.431
0.55	0.256	0.405	0.279	0.559	0.599	0.525
0.60	0.337	0.498	0.356	0.650	0.689	0.618
0.65	0.428	0.593	0.439	0.730	0.769	0.705
0.70	0.528	0.686	0.527	0.795	0.838	0.784
0.75	0.633	0.772	0.618	0.845	0.894	0.852
0.80	0.737	0.849	0.708	0.881	0.937	0.907
0.85	0.835	0.913	0.794	0.908	0.968	0.949
0.90	0.919	0.961	0.873	0.931	0.987	0.978
0.95	0.977	0.990	0.943	0.959	0.997	0.995
1.00	1.000	1.000	1.000	1.000	1.000	1.000

Table B - 4

Link Prob.	Error Eq. 31	Error Eq. 32
0.05	0.000	0.000
0.10	0.003	0.000
0.15	0.009	0.001
0.20	0.018	0.002
0.25	0.029	0.004
0.30	0.042	0.006
0.35	0.054	0.010
0.40	0.065	0.014
0.45	0.073	0.019
0.50	0.078	0.024
0.55	0.080	0.027
0.60	0.079	0.029
0.65	0.074	0.028
0.70	0.067	0.025
0.75	0.058	0.027
0.80	0.047	0.030
0.85	0.034	0.032
0.90	0.022	0.029
0.95	0.010	0.019
1.00	0.000	0.000

LIST OF REFERENCES

1. Gantz, Kenneth F., Lt. Col., The U.S. Air Force Report on the Ballistic Missile, Doubleday and Co., 1958.

2. Nuclear Weapons Phenomena and Characteristics, U.S. Office of Civil and Defense Mobilization, pp. 27-33, 1961.

3. Baran, P., "On Distributed Communication Networks," IEEE Trans. on Com. Tech., v. 12, pp. 1-9, March 1964.

4. Frank, H., "Vulnerability of Communication Networks," IEEE Trans. on Com Tech., v. 15, pp. 778-789, December 1967.

5. Fu, Y. "Applications of Topological Methods to Probabilistic Communication Network," IEEE Trans. on Com. Tech., v. 13, pp. 301-307, September 1965.

6. Solomonoff, R. and Rapaport, A., "Connectivity of Random Nets," Bulletin Math. Biophysics, v. 13, pp. 107-117, June 1951.

7. Chou, W. and Frank, H., "Survivable Communication Networks and Terminal Capacity Matrix," IEEE Trans. on Circuit Theory, v. 17, pp. 192-197, May 1970.

8. Shannon and Moore, "Reliable Circuits Using Less Reliable Relays," J. Franklin Inst., v. 262, pp. 191-208, September 1956.

9. Gomory, R. E. and Hu, T. C., "Multi-Terminal Network Flow," J. Soc. Indust. Appl. Math., v. 9, pp. 551-570, December 1961.

10. Ford and Fulkerson, "Maximal Flow Through a Network," Cand. J. Math., v. 8, pp. 399-404, 1956.

11. Chan, S. G., Fu, Y., and Chan, S. P., "On the Realization of Reability Functions of Probabilistic Communication Networks," Journal of Engineering Math., Wolters-Noordnoff Ltd., pp. 39-51, July 1967.

12. Rome Air Dev. Center Technical Report RADC-TR-60-159, Survival of Ground Electronics System from Nuclear Attack, by D. A. Benson, LT. K. D. Kirk, and Stanley M. Ostergen, October 1960.